STUDIES THE ARITHMATIC PROGRESSION RELATION WITH THE MAGIC SQUARES

Phan Văn Khái (Vietnam)

E-mail: khaiphanvan65@gmail.com

Abstract

How to create new arithmatic progression from the given arithmatic progression. Study the arithmatic progression relation with the magic squares. From there discover new miraculous characteristics.

Introduction: From two arithmetic progressions with the same common differences **d**, there is an arithmetic progression set according to rule: the effect of the sum of square or the sum of cube of each consecutive pair $(a_1;b_n)$, $(a_2;b_{n-1})$… will form the arithmetic progression. The pairing characteristics applied to magic squares by the method of borders [1] of author, the magic constant of the contours forms the arithmetic progression. The process of studying many new properties of magic squares was discovered.

Result:

Establishment of arithmetic progression

Square:

Give the arithmetic progression: $A_n = a_1, a_2, a_3, \ldots, a_n$.

$B_n = b_1, b_2, b_3, \ldots, b_n$ The common differences: **d**

Comment: $a_1+b_n = a_2+b_{n-1}=\ldots= a_n+b_1=C$

Establish: $(a_1^2+b_n^2)$, $(a_2^2+b_{n-1}^2)$, …, $(a_n^2+b_1^2)$.

The effect of two consecutive numbers, which will form a series of numbers:

$[(a_1^2+b_n^2)- (a_2^2+b_{n-1}^2)]$, $[(a_2^2+b_{n-1}^2)-(a_3^2+b_{n-2}^2)]$, …, $[(a_{n-1}^2+b_2^2)- (a_n^2+b_1^2)]$.

This is a arithmetic progression the common differences is **DS=2*2*d*d**

Cube:

Give the arithmetic progression: $A_n = a_1, a_2, a_3, \ldots, a_n$.

$B_n = b_1, b_2, b_3, \ldots, b_n$ The common differences: **d**

Establish: $(a_1^3+b_n^3)$, $(a_2^3+b_{n-1}^3)$, …, $(a_n^3+b_1^3)$.

The effect of two consecutive numbers, which will form a series of numbers:

$[(a_1^3+b_n^3)-(a_2^3+b_{n-1}^3)]$, $[(a_2^3+b_{n-1}^3)-(a_3^3+b_{n-2}^3)]$, …, $[(a_{n-1}^3+b_2^3)-(a_n^3+b_1^3)]$.

This is a arithmetic progression the common differences is **DC=2*3*d*d*(a_1+b_n)**

Prove

DS=2*2*d*d

$[(a_1^2+b_n^2)-(a_2^2+b_{n-1}^2)]-[(a_2^2+b_{n-1}^2)-(a_3^2+b_{n-2}^2)]=[(a_2^2+b_{n-1}^2)-(a_3^2+b_{n-2}^2)]-[(a_3^2+b_{n-2}^2)-(a_4^2+b_{n-3}^2)]=DS$

$(a_1^2+b_n^2)-2(a_2^2+b_{n-1}^2)+(a_3^2+b_{n-2}^2)=(a_1^2+b_n^2)-2[(a_1+d)^2+(b_n-d)^2]+[(a_1+2d)^2+(b_n-2d)^2]=a_1^2+b_n^2-2a_1^2-4a_1d-2d^2-2b_n^2+4b_nd-2d^2+a_1^2+4a_1d+4d^2+b_n^2-4b_nd+4d^2=4d^2=2*2*d*d$

Similar: $[(a_2^2+b_{n-1}^2)-(a_3^2+b_{n-2}^2)]-[(a_3^2+b_{n-2}^2)-(a_4^2+b_{n-3}^2)]=4d^2=2*2*d*d$

DC=2*3*d*d*(a_1+b_n)

$[(a_1^3+b_n^3)-(a_2^3+b_{n-1}^3)]-[(a_2^3+b_{n-1}^3)-(a_3^3+b_{n-2}^3)]=[(a_2^3+b_{n-1}^3)-(a_3^3+b_{n-2}^3)]-[(a_3^3+b_{n-2}^3)-(a_4^3+b_{n-3}^3)]=DC$

$(a_1^3+b_n^3)-2(a_2^3+b_{n-1}^3)+(a_3^3+b_{n-2}^3)=(a_1^3+b_n^3)-2[(a_1+d)^3+b_n-d)^3]+[(a_1+2d)^3+(b_n-2d)^3]=$

$a_1^3+b_n^3-2a_1^3-6a_1^2d-6a_1d^2-2d^3-2b_n^3+6b_n^2d-6b_nd^2+2d^3+a_1^3+6a_1^2d+12a_1d^2+8d^3+b_n^3-6b_n^2d+12b_nd^2-8d^3=6a_1d^2+6b_nd^2=6d^2(a_1+b_n)=2*3*d*d*(a_1+b_n)$

Similar: $[(a_2^3+b_{n-1}^3)-(a_3^3+b_{n-2}^3)]-[(a_3^3+b_{n-2}^3)-(a_4^3+b_{n-3}^3)]=2*3*d*d*(a_2+b_{n-1})=2*3*d*d*(a_1+b_n)$

Relationship between DS and DC

1) DC/DS=0 => $a_1=-b_n$

Then: DC=0, DS=2*2*d*d

2) DC+DS=2*d*d*[2+3(a_1+b_n)]=0 => $a_1+b_n=-2/3$

Then: DC= -DS= -2*2*d*d

Contact with the magic square

To the magic square odd:

Ex1: The common differences of the arithmetic progression: **d=0.5**

Suppose, starting with **A=10** (A is the average number)

Move to infinity:10,10.5,11, 11.5, 12, 12.5, 13, 13.5, 14, 14.5, 15, 15.5, 16, 16.5. 17, 17.5, 18, ...

Back to infinity: 10, 9.5, 9, 8.5, 8, 7.5, 7, 6.5, 6, 5.5, 5, 4.5, 4, 3.5, 3, 2.5, 2, …

Constant of the magic square 3x3, 5x5, 7x7, 9x9, 11x11, 13x13, 15x15,…will form a arithmetic progression the common differences **D=2*A=20**

10, 30, 50, 70, 90, 110, 130, 150, …

	3x3		30
11.5	8	10.5	30
9	10	11	30
9.5	12	8.5	30
30	30	30	30

90					9x9				
90	26.5	-8.5	-8	-7.5	-7	23.5	23	22.5	25.5
90	30	19.5	-1	-0.5	0	17	16.5	18.5	-10
90	29.5	22	14.5	4.5	5	12.5	13.5	-2	-9.5
90	29	21.5	16	11.5	8	10.5	4	-1.5	-9
90	-6	1	6	9	10	11	14	19	26
90	-5	2	7	9.5	12	8.5	13	18	25
90	-4.5	2.5	6.5	15.5	15	7.5	5.5	17.5	24.5
90	-4	1.5	21	20.5	20	3	3.5	0.5	24
90	-5.5	28.5	28	27.5	27	-3.5	-3	-2.5	-6.5
90	90	90	90	90	90	90	90	90	90

70				7x7			
70	19.5	-1	-0.5	0	17	16.5	18.5
70	22	14.5	4.5	5	12.5	13.5	-2
70	21.5	16	11.5	8	10.5	4	-1.5
70	1	6	9	10	11	14	19
70	2	7	9.5	12	8.5	13	18
70	2.5	6.5	15.5	15	7.5	5.5	17.5
70	1.5	21	20.5	20	3	3.5	0.5
70	70	70	70	70	70	70	70

		5x5			50
14.5	4.5	5	12.5	13.5	50
16	11.5	8	10.5	4	50
6	9	10	11	14	50
7	9.5	12	8.5	13	50
6.5	15.5	15	7.5	5.5	50
50	50	50	50	50	50

Ex2: **d=1/3**, **A=0** => D=0 (sum each pair equal 0)

0, 1/3, 2/3, 1, 4/3, 5/3, 2, 7/3 , 8/3, 3, 10/3, 11/3, 4, 13/3, 14/3, 5, 16/3, 17/3, 6, …

0, -1/3, -2/3, -1, -4/3, -5/3, -2, -7/3, -8/3, -3, -10/3, -11/3, -4, -13/3, -14/3, -5, -16/3, -17/3, -6, …

	3x3		0
1	-4/3	1/3	0
-2/3	0	2/3	0
-1/3	4/3	-1	0
0	0	0	0

0					9x9				
0	11	-37/3	-12	-35/3	-34/3	9	26/3	25/3	31/3
0	40/3	19/3	-22/3	-7	-20/3	14/3	13/3	17/3	-40/3
0	13	8	3	-11/3	-10/3	5/3	7/3	-8	-13
0	38/3	23/3	4	1	-4/3	1/3	-4	-23/3	-38/3
0	-32/3	-6	-8/3	-2/3	0	2/3	8/3	6	32/3
0	-10	-16/3	-2	-1/3	4/3	-1	2	16/3	10
0	-29/3	-5	-7/3	11/3	10/3	-5/3	-3	5	29/3
0	-28/3	-17/3	22/3	7	20/3	-14/3	-13/3	-19/3	28/3
0	-31/3	37/3	12	35/3	34/3	-9	-26/3	-25/3	-11
0	0	0	0	0	0	0	0	0	0

0				7x7			
0	19/3	-22/3	-7	-20/3	14/3	13/3	17/3
0	8	3	-11/3	-10/3	5/3	7/3	-8
0	23/3	4	1	-4/3	1/3	-4	-23/3
0	-6	-8/3	-2/3	0	2/3	8/3	6
0	-16/3	-2	-1/3	4/3	-1	2	16/3
0	-5	-7/3	11/3	10/3	-5/3	-3	5
0	-17/3	22/3	7	20/3	-14/3	-13/3	-19/3
0	0	0	0	0	0	0	0

		5x5			0
3	-11/3	-10/3	5/3	7/3	0
4	1	-4/3	1/3	-4	0
-8/3	-2/3	0	2/3	8/3	0
-2	-1/3	4/3	-1	2	0
-7/3	11/3	10/3	-5/3	-3	0
0	0	0	0	0	0

To the magic square even:

Ex1: **d=2**, **A=10**

Starting with 16 consecutive numbers: -5 -> 25

10 move to infinity:10, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45,

10 back to infinity: 10, 9, 7, 5, 3, 1, -1, -3, -5, -7, -9, -11, -13, -15, -17, -19, -21, -23, -25,

47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85, 87, ...

-27, -29, -31, -33, -35, -37, -39, -41, -43, -45, -47, -49, -51, -53, -55, -57, -59, -61, -63, -65, -67, ..

Constant of the magic square 4x4, 6x6, 8x8, 10x10, 12x12, 14x14, 16x16, … will form a arithmetic progression the common differences **D=2*A=20**

20, 40, 60, 80, 100, 120, 140, 160, 180, …

Since the four central cells do not have magic, start the sequence with 40

		4x4		40
-3	3	15	25	40
21	11	7	1	40
23	9	13	-5	40
-1	17	5	19	40
40	40	40	40	40

100					10x10					
100	-89	107	105	-77	-65	-63	81	79	77	-55
100	103	-53	71	69	51	49	-43	-37	-27	-83
100	101	67	-25	39	37	-13	29	-7	-47	-81
100	99	65	43	-3	3	15	25	-23	-45	-79
100	-75	61	41	21	11	7	1	-21	-41	95
100	93	-39	-15	23	9	13	-5	35	59	-73
100	-71	-35	-11	-1	17	5	19	31	55	91
100	-69	-33	27	-19	-17	33	-9	45	53	89
100	-67	47	-51	-49	-31	-29	63	57	73	87
100	75	-87	-85	97	85		-61	-59	-57	109
100	100	100	100	100	100	100	100	100	100	100

80				8x8				
80	-53	71	69	51	49	-43	-37	-27
80	67	-25	39	37	-13	29	-7	-47
80	65	43	-3	3	15	25	-23	-45
80	61	41	21	11	7	1	-21	-41
80	-39	-15	23	9	13	-5	35	59
80	-35	-11	-1	17	5	19	31	55
80	-33	27	-19	-17	33	-9	45	53
80	47	-51	-49	-31	-29	63	57	73
80	80	80	80	80	80	80	80	80

			6x6			60
-25	39	37	-13	29	-7	60
43	-3	3	15	25	-23	60
41	21	11	7	1	-21	60
-15	23	9	13	-5	35	60
-11	-1	17	5	19	31	60
27	-19	-17	33	-9	45	60
60	60	60	60	60	60	60

Ex2: **d=2$\sqrt{2}$**, **A=0** => D=0 (sum each pair equal 0)

Starting with 16 consecutive numbers: -15$\sqrt{2}$ -> 15$\sqrt{2}$

0, $\sqrt{2}$, 3$\sqrt{2}$, 5$\sqrt{2}$, 7$\sqrt{2}$, 9$\sqrt{2}$, 11$\sqrt{2}$, 13$\sqrt{2}$, 15$\sqrt{2}$, 17$\sqrt{2}$, 19$\sqrt{2}$, 21$\sqrt{2}$, 23$\sqrt{2}$,

0, -$\sqrt{2}$, -3$\sqrt{2}$, -5$\sqrt{2}$, -7$\sqrt{2}$, -9$\sqrt{2}$, -11$\sqrt{2}$, -13$\sqrt{2}$, -15$\sqrt{2}$, -17$\sqrt{2}$, -19$\sqrt{2}$, -21$\sqrt{2}$, -23$\sqrt{2}$,

$25\sqrt{2}$, $27\sqrt{2}$, $29\sqrt{2}$, $31\sqrt{2}$, $33\sqrt{2}$, $35\sqrt{2}$, $37\sqrt{2}$, $39\sqrt{2}$, $41\sqrt{2}$, $43\sqrt{2}$, $45\sqrt{2}$, $47\sqrt{2}$,.

$-25\sqrt{2}$, $-27\sqrt{2}$, $-29\sqrt{2}$, $-31\sqrt{2}$, $-33\sqrt{2}$, $-35\sqrt{2}$, $-37\sqrt{2}$, $-39\sqrt{2}$, $-41\sqrt{2}$, $-43\sqrt{2}$, $-45\sqrt{2}$, $-47\sqrt{2}$,

		4x4		0
$-13\sqrt{2}$	$-7\sqrt{2}$	$5\sqrt{2}$	$15\sqrt{2}$	0
$11\sqrt{2}$	$\sqrt{2}$	$-3\sqrt{2}$	$-9\sqrt{2}$	0
$13\sqrt{2}$	$-\sqrt{2}$	$3\sqrt{2}$	$-15\sqrt{2}$	0
$-11\sqrt{2}$	$7\sqrt{2}$	$-5\sqrt{2}$	$9\sqrt{2}$	0
0	0	0	0	0

0			6x6			
0	$-35\sqrt{2}$	$29\sqrt{2}$	$27\sqrt{2}$	$-23\sqrt{2}$	$19\sqrt{2}$	$-17\sqrt{2}$
0	$33\sqrt{2}$	$-13\sqrt{2}$	$-7\sqrt{2}$	$5\sqrt{2}$	$15\sqrt{2}$	$-33\sqrt{2}$
0	$31\sqrt{2}$	$11\sqrt{2}$	$\sqrt{2}$	$-3\sqrt{2}$	$-9\sqrt{2}$	$-31\sqrt{2}$
0	$-25\sqrt{2}$	$13\sqrt{2}$	$-\sqrt{2}$	$3\sqrt{2}$	$-15\sqrt{2}$	$25\sqrt{2}$
0	$-21\sqrt{2}$	$-11\sqrt{2}$	$7\sqrt{2}$	$-5\sqrt{2}$	$9\sqrt{2}$	$21\sqrt{2}$
0	$17\sqrt{2}$	$-29\sqrt{2}$	$-27\sqrt{2}$	$23\sqrt{2}$	$-19\sqrt{2}$	$35\sqrt{2}$
0	0	0	0	0	0	0

				8x8				0
$-63\sqrt{2}$	$61\sqrt{2}$	$59\sqrt{2}$	$-53\sqrt{2}$	$-47\sqrt{2}$	$41\sqrt{2}$	$39\sqrt{2}$	$-37\sqrt{2}$	0
$57\sqrt{2}$	$-35\sqrt{2}$	$29\sqrt{2}$	$27\sqrt{2}$	$-23\sqrt{2}$	$19\sqrt{2}$	$-17\sqrt{2}$	$-57\sqrt{2}$	0
$55\sqrt{2}$	$33\sqrt{2}$	$-13\sqrt{2}$	$-7\sqrt{2}$	$5\sqrt{2}$	$15\sqrt{2}$	$-33\sqrt{2}$	$-55\sqrt{2}$	0
$51\sqrt{2}$	$31\sqrt{2}$	$11\sqrt{2}$	$\sqrt{2}$	$-3\sqrt{2}$	$-9\sqrt{2}$	$-31\sqrt{2}$	$-51\sqrt{2}$	0
$-49\sqrt{2}$	$-25\sqrt{2}$	$13\sqrt{2}$	$-\sqrt{2}$	$3\sqrt{2}$	$-15\sqrt{2}$	$25\sqrt{2}$	$49\sqrt{2}$	0
$-45\sqrt{2}$	$-21\sqrt{2}$	$-11\sqrt{2}$	$7\sqrt{2}$	$-5\sqrt{2}$	$9\sqrt{2}$	$21\sqrt{2}$	$45\sqrt{2}$	0
$-43\sqrt{2}$	$17\sqrt{2}$	$-29\sqrt{2}$	$-27\sqrt{2}$	$23\sqrt{2}$	$-19\sqrt{2}$	$35\sqrt{2}$	$43\sqrt{2}$	0
$37\sqrt{2}$	$-61\sqrt{2}$	$-59\sqrt{2}$	$53\sqrt{2}$	$47\sqrt{2}$	$-41\sqrt{2}$	$-39\sqrt{2}$	$63\sqrt{2}$	0
0	0	0	0	0	0	0	0	0

					10x10					0
$-99\sqrt{2}$	$97\sqrt{2}$	$95\sqrt{2}$	$-87\sqrt{2}$	$-75\sqrt{2}$	$-73\sqrt{2}$	$71\sqrt{2}$	$69\sqrt{2}$	$67\sqrt{2}$	$-65\sqrt{2}$	0
$93\sqrt{2}$	$-63\sqrt{2}$	$61\sqrt{2}$	$59\sqrt{2}$	$-53\sqrt{2}$	$-47\sqrt{2}$	$41\sqrt{2}$	$39\sqrt{2}$	$-37\sqrt{2}$	$-93\sqrt{2}$	0
$91\sqrt{2}$	$57\sqrt{2}$	$-35\sqrt{2}$	$29\sqrt{2}$	$27\sqrt{2}$	$-23\sqrt{2}$	$19\sqrt{2}$	$-17\sqrt{2}$	$-57\sqrt{2}$	$-91\sqrt{2}$	0
$89\sqrt{2}$	$55\sqrt{2}$	$33\sqrt{2}$	$-13\sqrt{2}$	$-7\sqrt{2}$	$5\sqrt{2}$	$15\sqrt{2}$	$-33\sqrt{2}$	$-55\sqrt{2}$	$-89\sqrt{2}$	0
$-85\sqrt{2}$	$51\sqrt{2}$	$31\sqrt{2}$	$11\sqrt{2}$	$\sqrt{2}$	$-3\sqrt{2}$	$-9\sqrt{2}$	$-31\sqrt{2}$	$-51\sqrt{2}$	$85\sqrt{2}$	0
$83\sqrt{2}$	$-49\sqrt{2}$	$-25\sqrt{2}$	$13\sqrt{2}$	$-\sqrt{2}$	$3\sqrt{2}$	$-15\sqrt{2}$	$25\sqrt{2}$	$49\sqrt{2}$	$-83\sqrt{2}$	0
$-81\sqrt{2}$	$-45\sqrt{2}$	$-21\sqrt{2}$	$-11\sqrt{2}$	$7\sqrt{2}$	$-5\sqrt{2}$	$9\sqrt{2}$	$21\sqrt{2}$	$45\sqrt{2}$	$81\sqrt{2}$	0
$-79\sqrt{2}$	$-43\sqrt{2}$	$17\sqrt{2}$	$-29\sqrt{2}$	$-27\sqrt{2}$	$23\sqrt{2}$	$-19\sqrt{2}$	$35\sqrt{2}$	$43\sqrt{2}$	$79\sqrt{2}$	0
$-77\sqrt{2}$	$37\sqrt{2}$	$-61\sqrt{2}$	$-59\sqrt{2}$	$53\sqrt{2}$	$47\sqrt{2}$	$-41\sqrt{2}$	$-39\sqrt{2}$	$63\sqrt{2}$	$77\sqrt{2}$	0
$65\sqrt{2}$	$-97\sqrt{2}$	$-95\sqrt{2}$	$87\sqrt{2}$	$75\sqrt{2}$	$73\sqrt{2}$	$-71\sqrt{2}$	$-69\sqrt{2}$	$-67\sqrt{2}$	$99\sqrt{2}$	0
0	0	0	0	0	0	0	0	0	0	0

Summation the magic squares

(There are many magic squares [2]. Only true with two magic squares set up the same)

					10
11	-9	-8	7	9	10
14	5	-2	3	-10	10
-6	0	2	4	10	10
-4	1	6	-1	8	10
-5	13	12	-3	-7	10
10	10	10	10	10	10

From -10 to 14, d=1

+

					15
21	-19	-17	13	17	15
27	9	-5	5	-21	15
-13	-1	3	7	19	15
-9	1	11	-3	15	15
-11	25	23	-7	-15	15
15	15	15	15	15	15

From -21 to 27, d=2

=

					25
32	-28	-25	20	26	25
41	14	-7	8	-31	25
-19	-1	5	11	29	25
-13	2	17	-4	23	25
-16	38	35	-10	-22	25
25	25	25	25	25	25

From -31 to 41, d=3

						111
1	33	32	7	28	10	111
35	12	15	21	26	2	111
34	24	19	17	14	3	111
6	25	18	20	11	31	111
8	13	22	16	23	29	111
27	4	5	30	9	36	111
111	111	111	111	111	111	111

From 1 to 36, d=1

+

						555
5	165	160	35	140	50	555
175	60	75	105	130	10	555
170	120	95	85	70	15	555
30	125	90	100	55	155	555
40	65	110	80	115	145	555
135	20	25	150	45	180	555
555	555	555	555	555	555	555

From 5 to 180, d=5

=

						666
6	198	192	42	168	60	666
210	72	90	126	156	12	666
204	144	114	102	84	18	666
36	150	108	120	66	186	666
48	78	132	96	138	174	666
162	24	30	180	54	216	666
666	666	666	666	666	666	666

From 6 to 216, d=6

					10
11	-9	-8	7	9	10
14	5	-2	3	-10	10
-6	0	2	4	10	10
-4	1	6	-1	8	10
-5	13	12	-3	-7	10
10	10	10	10	10	10

From -10 to 14, d=1

+

					15
21	-19	-17	13	17	15
27	5	7	-3	-21	15
-13	-5	3	11	19	15
-9	9	-1	1	15	15
-11	25	23	-7	-15	15
15	15	15	15	15	15

From -21 to 27, d=2

=

					25
32	-28	-25	20	26	25
41	10	5	0	-31	25
-19	-5	5	15	29	25
-13	10	5	0	23	25
-16	38	35	-10	-22	25
25	25	25	25	25	25

The numbers are cluttered

Summation the reversible magic squares

					10
11	-9	-8	7	9	10
14	5	-2	3	-10	10
-6	0	2	4	10	10
-4	1	6	-1	8	10
-5	13	12	-3	-7	10
10	10	10	10	10	10

From -10 to 14, d=1

+

Reversible

					15
-15	25	23	-7	-11	15
-21	-3	11	1	27	15
19	7	3	-1	-13	15
15	5	-5	9	-9	15
17	-19	-17	13	21	15
15	15	15	15	15	15

From -21 to 27, d=2

=

					25
-4	16	15	0	-2	25
-7	2	9	4	17	25
13	7	5	3	-3	25
11	6	1	8	-1	25
12	-6	-5	10	14	25
25	25	25	25	25	25

From -7 to 17, d=1

						111
1	33	32	7	28	10	111
35	12	15	21	26	2	111
34	24	19	17	14	3	111
6	25	18	20	11	31	111
8	13	22	16	23	29	111
27	4	5	30	9	36	111
111	111	111	111	111	111	111

From 1 to 36, d=1

+

Reversible

						555
180	20	25	150	45	135	555
10	125	110	80	55	175	555
15	65	90	100	115	170	555
155	60	95	85	130	30	555
145	120	75	105	70	40	555
50	165	160	35	140	5	555
555	555	555	555	555	555	555

From 5 to 180, d=5

=

						666
181	53	57	157	73	145	666
45	137	125	101	81	177	666
49	89	109	117	129	173	666
161	85	113	105	141	61	666
153	133	97	121	93	69	666
77	169	165	95	149	41	666
666	666	666	666	666	666	666

From 41 to 181, d=4

					125
43	3	5	35	39	125
49	31	17	27	1	125
9	21	25	29	41	125
13	23	33	19	37	125
11	47	45	15	7	125
125	125	125	125	125	125

From 1 to 49, d=2

+

Reversible

					130
8	48	46	16	12	130
2	20	34	24	50	130
42	30	26	22	10	130
38	28	18	32	14	130
40	4	6	36	44	130
130	130	130	130	130	130

From 2 to 50, d=2

=

					255
51	51	51	51	51	255
51	51	51	51	51	255
51	51	51	51	51	255
51	51	51	51	51	255
51	51	51	51	51	255
255	255	255	255	255	255

From 51 to 51, d=0

Summation of two magic squares have average number opposite

					5
10	-11	-9	7	8	5
12	4	-3	2	-10	5
-7	-1	1	3	9	5
-4	0	5	-2	6	5
-6	13	11	-5	-8	5
5	5	5	5	5	5

From -11 to 13, d=1

+

					-5
8	-13	-11	5	6	-5
10	2	-5	0	-12	-5
-9	-3	-1	1	7	-5
-6	-2	3	-4	4	-5
-8	11	9	-7	-10	-5
-5	-5	-5	-5	-5	-5

From -13 to 11, d=1

=

					0
18	-24	-20	12	14	0
22	6	-8	2	-22	0
-16	-4	0	4	16	0
-10	-2	8	-6	10	0
-14	24	20	-12	-18	0
0	0	0	0	0	0

From -24 to 24, d=2

					5
10	-11	-9	7	8	5
12	4	-3	2	-10	5
-7	-1	1	3	9	5
-4	0	5	-2	6	5
-6	13	11	-5	-8	5
5	5	5	5	5	5

From -11 to 13, d=1

+

Reversible

					-5
-10	11	9	-7	-8	-5
-12	-4	3	-2	10	-5
7	1	-1	-3	-9	-5
4	0	-5	2	-6	-5
6	-13	-11	5	8	-5
-5	-5	-5	-5	-5	-5

From -13 to 11, d=1

=

					0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

From 0 to 0, d=0

						6
-34	30	28	-22	20	-16	6
34	-12	-6	6	16	-32	6
32	12	2	-2	-8	-30	6
-24	14	0	4	-14	26	6
-20	-10	8	-4	10	22	6
18	-28	-26	24	-18	36	6
6	6	6	6	6	6	6

From -34 to 36, d=2

+

						-6
-36	28	26	-24	18	-18	-6
32	-14	-8	4	14	-34	-6
30	10	0	-4	-10	-32	-6
-26	12	-2	2	-16	24	-6
-22	-12	6	-6	8	20	-6
16	-30	-28	22	-20	34	-6
-6	-6	-6	-6	-6	-6	-6

From -36 to 34, d=2

=

						0
-70	58	54	-46	38	-34	0
66	-26	-14	10	30	-66	0
62	22	2	-6	-18	-62	0
-50	26	-2	6	-30	50	0
-42	-22	14	-10	18	42	0
34	-58	-54	46	-38	70	0
0	0	0	0	0	0	0

From -70 to 70, d=4

						6
-34	30	28	-22	20	-16	6
34	-12	-6	6	16	-32	6
32	12	2	-2	-8	-30	6
-24	14	0	4	-14	26	6
-20	-10	8	-4	10	22	6
18	-28	-26	24	-18	36	6
6	6	6	6	6	6	6

From -34 to 36, d=2

+

Reversible

						-6
34	-30	-28	22	-20	16	-6
-34	12	6	-6	-16	32	-6
-32	-12	-2	2	8	30	-6
24	-14	0	-4	14	-26	-6
20	10	-8	4	-10	-22	-6
-18	28	26	-24	18	-36	-6
-6	-6	-6	-6	-6	-6	-6

From -36 to 34, d=2

=

						0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

From 0 to 0, d=0

Features of magic square 4x4

1) There are always twins

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

>

a	c	b	d
i	k	j	l
e	g	f	h
m	o	n	p

2) If: b+n=c+o or e+h=i+l, then: Born four. Ex: b+n=c+o

a	c	b	d
i	k	j	l
e	g	f	h
m	o	n	p

<

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

<->

a	c	b	d
e	f	g	h
i	j	k	l
m	o	n	p

>

a	b	c	d
i	k	j	l
e	g	f	h
m	n	o	p

3) Each magic square has a reversible magic square

1	15	6	12
14	4	9	7
11	5	16	2
8	10	3	13

>

16	2	11	5
3	13	8	10
6	12	1	15
9	7	14	4

Sum of corresponding cells equal 17

4) If a magic square has characteristic: any square 2x2 equal 34, then twin brother has characteristic: 4 edge of the square next equal 17. Ex:

1	15	6	12
14	4	9	7
11	5	16	2
8	10	3	13

>

1	6	15	12
11	16	5	2
14	9	4	7
8	3	10	13

1+14+15+4=15+4+6+9=…=7+2+9+16=34

4 edge of the square next: 6+11=14+3=10+7=2+15=17

There are 90/880 magic squares [3] of this type

5) Born four: if sum 2 edge of the square next equal 34 (h+c+b+e or c+h+l+o equal 34), then sum of square of 2 diagonal equal ($a^2+f^2+k^2+p^2=d^2+g^2+j^2+m^2$). Ex:

1	15	2	16
13	10	7	4
12	3	14	5
8	6	11	9

<

1	2	15	16
12	14	3	5
13	7	10	4
8	11	6	9

<->

1	2	15	16
13	14	3	4
12	7	10	5
8	11	6	9

>

1	15	2	16
12	10	7	5
13	3	14	4
8	6	11	9

5+15+2+12=34, then $1^2+14^2+10^2+9^2=16^2+3^2+7^2+8^2$

There are 304/880 magic squares of this type

6) There are 4 cases outside the group also have the above characteristic. Sum of square of 2 diagonals equal and sum of cube of 2 diagonals equal.

2	13	16	3
11	8	5	10
7	12	9	6
14	1	4	15

>

2	16	13	3
7	9	12	6
11	5	8	10
14	4	1	15

2	11	16	5
13	8	3	10
7	14	9	4
12	1	6	15

>

2	16	11	5
7	9	14	4
13	3	8	10
12	6	1	15

3	16	6	9
10	5	15	4
13	2	12	7
8	11	1	14

>

3	6	16	9
13	12	2	7
10	15	5	4
8	1	11	14

5	16	4	9
10	3	15	6
11	2	14	7
8	13	1	12

>

5	4	16	9
11	14	2	7
10	15	3	6
8	1	13	12

$(1^2+2^2+\ldots+16^2)/4=374$; $(1^3+2^3+\ldots+16^3)/4=4624$. They are magic constants

7) Sum of 4 squares in the corner = sum of 4 squares in the center = sum of 4 squares green = sum of 4 squares blue = 34

8) Sum of 2 squares between of column or row = sum of 2 squares in the corner opposite

9) Sum of 2 squares in the center on this diagonal = sum of 2 squares in the corner on that diagonal

Application exercises

Please fill in the numbers 1 to 16 in the square 4x4 to satisfy all of the following conditions:

- Sum of rows, columns and diagonals are equal.
- Sum of square of numbers on two diagonals is equal.
- Sum of cube of numbers on two diagonals is equal.

Illustration:

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

Condition 1:
a+b+c+d=e+f+g+h=i+j+k+l=m+n+o+p=a+e+i+m=b+f+j+n=c+g+k+o=d+h+l+p=a+f+k+p=d+g+j+m=A

Condition 2: a^2+f^2+k^2+p^2=d^2+g^2+j^2+m^2=B

Condition 3: a^3+f^3+k^3+p^3=d^3+g^3+j^3+m^3=C

The answer

Use excel

Step 1: List numbers groups: 1+2+15+16
1+3+14+16
…………..
7+6+10+11
7+8+9+10 There are 86 groups

Step 2: Sum of square of numbers on the numbers groups
Sum of cube of numbers on the numbers groups

Step 3: in the column square, ranked from small to large. Find the same results (3 cases)

a) 3+7+10+14 -> 3^2+7^2+10^2+14^2=354
-> 3^3+7^3+10^3+14^3=4114
4+5+12+13 -> 4^2+5^2+12^2+13^2=354
-> 4^3+5^3+12^3+13^3=4114 No exist

b) 2+8+9+15 -> 2^2+8^2+9^2+15^2=374
-> 2^3+8^3+9^3+15^3=4624
3+5+12+14 -> 3^2+5^2+12^2+14^2=374
-> 3^3+5^3+12^3+14^3=4624 Exist

c) 1+6+11+16 -> 1^2+6^2+11^2+16^2=414
-> 1^3+6^3+11^3+16^3=5644
2+4+13+15 -> 2^2+4^2+13^2+15^2=414
-> 2^3+4^3+13^3+15^3=5644 No exist

Step 4: From result (b), make swap find out 4 results
Step 5: Characteristic 1, inferred 8 results as part of the theory.

Composite magic squares [4]

Harry White's Method

Square 12x12:

Magic square 12x12												870
116	128	125	113	4	16	13	1	84	96	93	81	870
123	119	122	118	11	7	10	6	91	87	90	86	870
117	121	120	124	5	9	8	12	85	89	88	92	870
126	114	115	127	14	2	3	15	94	82	83	95	870
36	48	45	33	68	80	77	65	100	112	109	97	870
43	39	42	38	75	71	74	70	107	103	106	102	870
37	41	40	44	69	73	72	76	101	105	104	108	870
46	34	35	47	78	66	67	79	110	98	99	111	870
52	64	61	49	132	144	141	129	20	32	29	17	870
59	55	58	54	139	135	138	134	27	23	26	22	870
53	57	56	60	133	137	136	140	21	25	24	28	870
62	50	51	63	142	130	131	143	30	18	19	31	870
870	870	870	870	870	870	870	870	870	870	870	870	870

Magic square 12x12												870
8	1	6	125	118	123	134	127	132	35	28	33	870
3	5	7	120	122	124	129	131	133	30	32	34	870
4	9	2	121	126	119	130	135	128	31	36	29	870
107	100	105	62	55	60	53	46	51	80	73	78	870
102	104	106	57	59	61	48	50	52	75	77	79	870
103	108	101	58	63	56	49	54	47	76	81	74	870
71	64	69	98	91	96	89	82	87	44	37	42	870
66	68	70	93	95	97	84	86	88	39	41	43	870
67	72	65	94	99	92	85	90	83	40	45	38	870
116	109	114	17	10	15	26	19	24	143	136	141	870
111	113	115	12	14	16	21	23	25	138	140	142	870
112	117	110	13	18	11	22	27	20	139	144	137	870
870	870	870	870	870	870	870	870	870	870	870	870	870
Square 3x3: 1 2 3 4 5 6 7 8 9												
10 11 12 13 14 15 16 17 18												

Phan Van Khai's Method

Square 8x8:

Magic square 8x8								260
6	63	1	60	14	11	53	52	260
3	58	8	61	55	50	16	9	260
64	5	59	2	12	13	51	54	260
57	4	62	7	49	56	10	15	260
22	47	17	44	30	27	37	36	260
19	42	24	45	39	34	32	25	260
48	21	43	18	28	29	35	38	260
41	20	46	23	33	40	26	31	260
260	260	260	260	260	260	260	260	260

4 squares 4x4 are equal value

Magic square 8x8								260
4	57	12	49	15	54	7	62	260
13	56	5	64	2	59	10	51	260
20	41	28	33	31	38	23	46	260
29	40	21	48	18	43	26	35	260
34	27	42	19	45	24	37	32	260
47	22	39	30	36	25	44	17	260
50	11	58	3	61	8	53	16	260
63	6	55	14	52	9	60	1	260
260	260	260	260	260	260	260	260	260

Any square 2x2 has a value of half the value of the square 8x8

Square 11x11:

Magic square 11x11											671
112	1	2	3	4	9	109	108	107	106	110	671
117	91	84	89	28	21	26	73	66	71	5	671
116	86	88	90	23	25	27	68	70	72	6	671
115	87	92	85	24	29	22	69	74	67	7	671
114	46	39	44	64	57	62	82	75	80	8	671
11	41	43	45	59	61	63	77	79	81	111	671
17	42	47	40	60	65	58	78	83	76	105	671
18	55	48	53	100	93	98	37	30	35	104	671
19	50	52	54	95	97	99	32	34	36	103	671
20	51	56	49	96	101	94	33	38	31	102	671
12	121	120	119	118	113	13	14	15	16	10	671
671	671	671	671	671	671	671	671	671	671	671	671

Square 9x9 same Harry White

Square 12x12:

Magic square 12x12												870
6	143	1	140	14	135	9	132	22	127	17	124	870
3	138	8	141	11	130	16	133	19	122	24	125	870
144	5	139	2	136	13	131	10	128	21	123	18	870
137	4	142	7	129	12	134	15	121	20	126	23	870
46	103	41	100	38	111	33	108	30	119	25	116	870
43	98	48	101	35	106	40	109	27	114	32	117	870
104	45	99	42	112	37	107	34	120	29	115	26	870
97	44	102	47	105	36	110	39	113	28	118	31	870
54	95	49	92	62	87	57	84	70	79	65	76	870
51	90	56	93	59	82	64	85	67	74	72	77	870
96	53	91	50	88	61	83	58	80	69	75	66	870
89	52	94	55	81	60	86	63	73	68	78	71	870
870	870	870	870	870	870	870	870	870	870	870	870	870

The squares 4x4 are equal value, so put in any location

Magic square 12x12												870
113	1	81	126	14	94	127	15	95	116	4	84	870
33	65	97	46	78	110	47	79	111	36	68	100	870
49	129	17	62	142	30	63	143	31	52	132	20	870
124	12	92	119	7	87	118	6	86	121	9	89	870
44	76	108	39	71	103	38	70	102	41	73	105	870
60	140	28	55	135	23	54	134	22	57	137	25	870
120	8	88	123	11	91	122	10	90	117	5	85	870
40	72	104	43	75	107	42	74	106	37	69	101	870
56	136	24	59	139	27	58	138	26	53	133	21	870
125	13	93	114	2	82	115	3	83	128	16	96	870
45	77	109	34	66	98	35	67	99	48	80	112	870
61	141	29	50	130	18	51	131	19	64	144	32	870
870	870	870	870	870	870	870	870	870	870	870	870	870

Square 3x3: 1 17 33 49 65 81 97 113 129

2 18 34 50 66 82 98 114 130

Magic square 12x12												870
113	1	81	127	15	95	118	6	86	124	12	92	870
33	65	97	47	79	111	38	70	102	44	76	108	870
49	129	17	63	143	31	54	134	22	60	140	28	870
120	8	88	122	10	90	115	3	83	125	13	93	870
40	72	104	42	74	106	35	67	99	45	77	109	870
56	136	24	58	138	26	51	131	19	61	141	29	870
123	11	91	117	5	85	128	16	96	114	2	82	870
43	75	107	37	69	101	48	80	112	34	66	98	870
59	139	27	53	133	21	64	144	32	50	130	18	870
126	14	94	116	4	84	121	9	89	119	7	87	870
46	78	110	36	68	100	41	73	105	39	71	103	870
62	142	30	52	132	20	57	137	25	55	135	23	870
870	870	870	870	870	870	870	870	870	870	870	870	870

Any square 6x6 has a value is three times the value of the square12x12

Magic square 12x12												870
1	142	141	140	9	2	19	124	123	122	27	20	870
6	16	133	11	130	139	24	34	31	113	112	121	870
138	13	128	18	131	7	120	115	110	36	29	25	870
137	134	15	129	12	8	119	32	33	111	114	26	870
10	127	14	132	17	135	28	109	116	30	35	117	870
143	3	4	5	136	144	125	21	22	23	118	126	870
37	106	105	104	45	38	55	88	87	86	63	56	870
42	52	97	47	94	103	60	70	67	77	76	85	870
102	49	92	54	95	43	84	79	74	72	65	61	870
101	98	51	93	48	44	83	68	69	75	78	62	870
46	91	50	96	53	99	64	73	80	66	71	81	870
107	39	40	41	100	108	89	57	58	59	82	90	870
870	870	870	870	870	870	870	870	870	870	870	870	870

4 squares 6x6 are equal value, 4 squares 4x4 are equal value

Magic square 12x12												870
1	142	141	140	139	138	123	8	9	10	17	2	870
134	23	120	119	118	117	105	31	33	35	24	11	870
12	29	70	79	65	76	62	59	85	84	116	133	870
13	115	67	74	72	77	87	82	64	57	30	132	870
131	113	80	69	75	66	60	61	83	86	32	14	870
130	34	73	68	78	71	81	88	58	63	111	15	870
16	109	54	95	49	92	46	43	101	100	36	129	870
127	37	51	90	56	93	103	98	48	41	108	18	870
19	38	96	53	91	50	44	45	99	102	107	126	870
20	106	89	52	94	55	97	104	42	47	39	125	870
124	121	25	26	27	28	40	114	112	110	122	21	870
143	3	4	5	6	7	22	137	136	135	128	144	870
870	870	870	870	870	870	870	870	870	870	870	870	870
4 squares 4x4 are equal value												

Discussion:

1) If the common differences of A_n is d_1, of B_n is d_2 then $DS=2*(d_1^2+d_2^2)$, when $d_1=d_2$ then DS=2*2*d*d. And DC: there is no arithmetic progression when $d_1 \neq d_2$.
2) Magic squares depend only on the average number A and not depend on the common differences **d**. The arithmetic progression of the magic square odd and the arithmetic progression of the magic square even have the same average number A is interspersed. As above example A=10, the arithmetic progression of the magic square odd: 10, 30, 50, 70, 90, 110, …, the arithmetic progression of the magic square even: 20, 40, 60, 80, 100, 120, … When knowing the average number is A, then constant of the magic square 1x1, 2x2, 3x3, 4x4, 5x5, …, nxn is 1*A, 2*A (non magic), 3*A, 4*A, 5*A, …, n*A.
3) Magic squares are established with all arithmetic progression, the common diference **d** are real numbers.
4) M_1 is magic square of arithmetic progression A_1, common differences d_1. M_2 is magic square of arithmetic progression A_2, common differences d_2. Set up the same. Then $M_1+M_2=M_3$ and $d_3=d_1+d_2$. If M'_2 is reversible of M_2 then $M_1+M'_2=M_3$ and $d_3= d_2-d_1$.
5) Two magic squares have average number opposite then constant of M_3 equal 0.
6) Magic squares have many interesting characteristics that need to be explored.

Conclusion:

Magic squares are closely related to the arithmetic progression.

Magic squares have many interesting attributes.

References:

1. "Magic square" https://en.wikipedia.org/wiki/Magic_square
2. Trump, Walter "Magic Square" http://www.trump.de/magic-squares/
3. Heinz, Harvey "Magic Square" http://magic-squares.net/
4. White, Harry "Magic Square" http://budshaw.ca/MagicSquares.html